FLOW NEAR THE STAGNATION POINT OF AN OBSTACLE WASHED BY A TWO-DIMENSIONAL SUBSONIC STREAM

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An investigation is made of two-dimensional steady flow of a viscous incompressible fluid in the vicinity of the stagnation point of an infinite step, in flow of a subsonic free jet. The velocity profile of the undisturbed flow far from the step takes the form of a power series in the coordinate along the step surface. Results of numerical solution are presented.

Statement of the Problem and Basic Equations. In the coordinate system XOY, where X is directed along the obstacle and Y normal to it, an incident stream is bounded by the two-dimensional obstacle Y = 0 with a stagnation point X = Y = 0. To analyze the flow at any other stagnation point we use the continuity equation and the Navier–Stokes equation in the form of the vorticity transport equation [1]:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0,$$
$$v_x \frac{\partial \omega}{\partial x} + v_y \frac{\partial \omega}{\partial y} = \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}, \qquad (1)$$

$$\omega = \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}; \quad x,\ y = (X,\ Y)\sqrt{\frac{\beta}{\nu}},$$
$$v_x,\ v_y = \frac{V_X,\ V_Y}{\sqrt{\beta\nu}}, \quad \omega = \frac{\Omega(X,\ Y)}{\beta}, \qquad (2)$$

where β is the velocity gradient at the stagnation point; ν is the coefficient of kinematic viscosity; and V_X, V_Y, $\Omega(X, Y)$ are the velocity components in the vortex at the physical coordinates X and Y.

We choose the following characteristic zones in the flow field near the obstacle:

1) the undisturbed flow whose velocity v is known and given by a power series containing only even powers of x:

$$v = -\sum_{n=0}^{\infty} (-1)^n a_{2n} x^{2n}, \qquad (3)$$

where a_{2n} are given coefficients determining the undisturbed flow of the free jet;

2) the inviscid zone of interaction of the flow with the obstacle, where the velocity component v_y varies linearly with the coordinate y:

$$v_{y\infty} = -\sum_{n=0}^{\infty} (-1)^n a_{2n} x^{2n} y; \qquad (4)$$

3) the viscous wall layer at the obstacle, where, in analogy with the series (4), we take for v_y the expansion:

$$v_y = -\sum_{n=0}^{\infty} (-1)^n a_{2n} x^{2n} f_{2n}. \qquad (5)$$

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Using Eq. (5), we obtain the following expression for v_x from Eq. (1):

$$v_x = \sum_{n=0}^{\infty} (-1)^n \frac{a_{2n}}{2n+1} x^{2n+1} f'_{2n}. \tag{6}$$

The vorticity ω and the tangential stress at the wall τ_w are determined by the following power series in x:

$$\omega = -\sum_{n=0}^{\infty} (-1)^n a_{2n} x^{2n-1} \left(\frac{f''_{2n}}{2n+1} x^2 + 2n f_{2n} \right), \tag{7}$$

$$\tau_w = \sum_{n=0}^{\infty} (-1)^n \frac{a_{2n}}{2n+1} x^{2n+1} f''_{2n}, \tag{8}$$

where $\tau_w = \tau_w(X)/\mu\beta$, $\mu = \rho\nu$. Taking account of Eqs. (5)-(7), the vorticity transport equation (2) can be written as:

$$\sum_{n=0}^{\infty} (-1)^n a_{2n} x^{2n-3} \left[\frac{x^4}{2n+1} f^{IV}_{2n} + 4n x^2 f''_{2n} + 4n(2n-1)(n-1) f_{2n} \right]$$
$$+ \sum_{n,k}^{\infty} (-1)^{n+k} a_{2n} a_{2k} x^{2(n+k)-1} \left[x^2 \left(\frac{f_{2n} f'''_{2k}}{2k+1} - \frac{f'_{2n} f''_{2k}}{2n+1} \right) \right.$$
$$\left. + 2k \left(f_{2n} f''_{2k} - \frac{2k-1}{2n+1} f'_{2n} f_{2k} \right) \right] = 0. \tag{9}$$

By equating coefficients of the same powers of x, we obtain from Eq. (9) a system of differential equations to determine the functions f_{2n} which depend only on y. The number of equations is determined by the number of terms evaluated in the above series. It is clear that as one moves away from the stagnation point the number of terms should be increased. A numerical experiment shows that in analyzing the flow near the stagnation point 4 terms of the series is sufficient for a satisfactory solution. For $n \le 3$ the system has the form

$$\begin{aligned}
& a_0 f_0^{IV} + a_0^2 f_0 f_0''' - a_0^2 f_0' f_0'' - 2a_0 a_2 f_0 f_2' + 2a_0 a_2 f_0' f_2 - 4a_2 f_2'' + 24 a_4 f_4 = 0, \\
& a_2 f_2^{IV} + a_0 a_2 f_0 f_2''' - 3a_0 a_2 f_0' f_2'' - a_0 a_2 f_0'' f_2' - 4a_2^2 f_2 f_2' + 3a_0 a_2 f_0''' f_2 \\
& \quad + 36 a_0 a_4 f_0' f_4 - 12 a_0 a_4 f_0 f_4' - 24 a_4 f_4'' + 360 a_6 f_6 = 0, \\
& a_4 f_4^{IV} + a_0 a_4 f_0 f_4''' - 5a_0 a_4 f_0' f_4'' - a_0 a_4 f_0'' f_4' - 18 a_2 a_4 f_2 f_4' \\
& \quad + 10 a_2 a_4 f_2' f_4 + 5a_0 a_4 f_0''' f_4 + \frac{5}{3} a_2^2 f_2 f_2''' - \frac{5}{3} a_2^2 f_2' f_2'' \\
& \quad - 30 a_0 a_6 f_0 f_6' + 150 a_0 a_6 f_0' f_6 - 60 a_6 f_6'' = 0, \\
& a_6 f_6^{IV} + a_0 a_6 f_0 f_6 - 7a_0 a_6 f_0' f_6'' - a_0 a_6 f_0'' f_6' - 40 a_2 a_6 f_2 f_6' \\
& \quad + 7 a_0 a_6 f_0''' f_6 + 56 a_2 a_6 f_2' f_6 - \frac{56}{5} a_4^2 f_4 f_4' - \frac{7}{5} a_2 a_4 f_2 f_4''' \\
& \quad - \frac{7}{3} a_2 a_4 f_2' f_4'' + \frac{7}{3} a_2 a_4 f_2''' f_4 - \frac{7}{5} a_2 a_4 f_2'' f_4' = 0.
\end{aligned} \tag{10}$$

For solution of this system the boundary conditions to be used are:

$$\begin{aligned}
& y = 0, \quad f_{2n} = f'_{2n} = 0, \\
& y \to \infty, \quad f'_{2n} = 1, \quad f''_{2n} = 0 \quad (n = 0,\ 1,\ 2,\ 3).
\end{aligned} \tag{11}$$

Numerical Solution. The system of equations (10), with boundary conditions (11), was solved numerically on the BÉSM-4 computer by the method of successive approximations. Integration was performed by the Runge–Kutta method with accuracy to 10^{-5}. It should be noted that the zeroth approximation gives the well-known Himenitz solution [1] for two-dimensional flow near a stagnation point. In subsequent approximations the values of the functions determined from the previous approximations are taken as constants at each step of integration. The latter assumption does not affect the convergence process, as was verified by doubling the integration step size. The search for the unknown conditions at the wall, i.e., $f''(0)$ and $f'''(0)$, was performed by Newton's method. The functions $[f'(\infty)-1]$ and $f''(\infty)$ were expanding in Taylor series in the vicinity of the roots $f''(0)$ and $f'''(0)$ using approximate values in the Taylor series for the partial

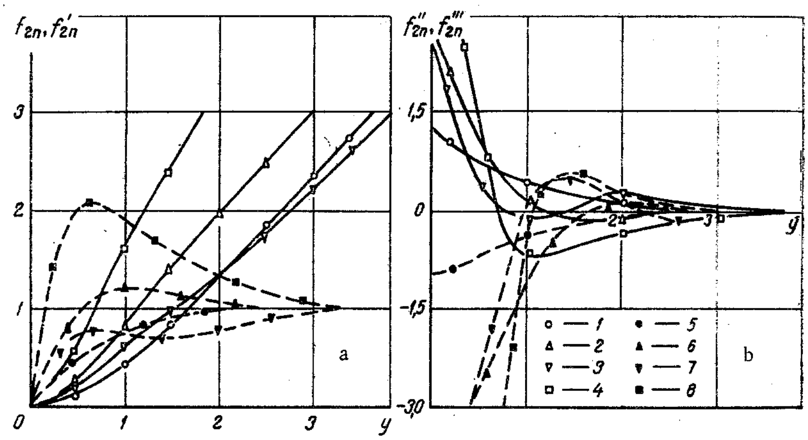

Fig. 1. Results of calculation of functions f_{2n} and f'_{2n} (a): plot of 1) f_0; 2) f_2; 3) f_4; 4) f_6; 5) f'_0; 6) f'_2; 7) f'_4; 8) f'_6, and f''_{2n} and f'''_{2n} (b): plot of 1) f''_0; 2) f'_2; 3) f''_4; 4) f''_6; 5) f'''_0; 6) f'''_2; 7) f'''_4; 8) f'''_6.

TABLE 1. Results of Solution of Eq. (10)

n	0	1	2	3
$f''_{2n}(0)$	1,2326	2,8155	2,7652	7,7502
$f'''_{2n}(0)$	—1	—4,0041	—6,1493	—18,364

derivatives, obtained by polynomial interpolation from the Gregory–Newton formula [2], allowing for differences to fourth order. The system of algebraic equations in f"(0) and f'''(0) thus obtained was solved by an iterative method. For the first integration the values of f"(0) and f'''(0) were assigned arbitrarily from the expected range of the exact values.

The velocity profile V of the unperturbed jet flow was taken, from the experimental data of [3], in the form

$$\frac{V}{V_m} = e^{-k\left(\frac{X}{X_{0,5}}\right)^2}, \tag{12}$$

where $k = 0.69$; V_m is the maximum value of the unperturbed flow at $X = 0$; $X_{0.5}$ is the distance of the obstacle, where $V = 0.5\ V_m$. In the region of interaction of the jet with the obstacle a linear law was used for the variation of normal velocity component, i.e.,

$$V_{Y_\infty} = -\frac{V_m}{Y_\infty} Y e^{-k\left(\frac{X}{X_{0,5}}\right)^2},$$

where Y_∞ is the distance from the obstacle to the boundary of the interaction region, where Eq. (12) is valid.

Introducing the velocity gradient at the stagnation point $\beta = V_m/V_\infty$, the resulting expression can be written in dimensionless form as

$$v_{y_\infty} = -ye^{-k\left(\frac{x}{x_{0,5}}\right)^2}. \tag{13}$$

Comparing Eqs. (13) and (4), we obtain

$$e^{-k\left(\frac{x}{x_{0,5}}\right)^2} = \sum_{n=0}^{\infty} (-1)^{2n} a_{2n} x^{2n},$$

whence it follows that $a_0 = 1$, $a_2 = k/x_{0.5}^2$, $a_4 = k^2/2x_{0.5}^4$, $a_6 = k^2/6x_{0.5}^6$, etc.

From the data in [3] for $x_{0.5}$, the exit section in the series coefficients, we can obtain

$$X_{0.5} = \frac{B_a}{2} + 0.0848Y,$$

where B_a is the width of the nozzle exit section; Y is the distance from the nozzle exit to the section considered, corresponding to the boundary of the jet–step interaction region, and $Y = Y_0 - Y_\infty$, where Y_0 is the distance of the obstacle from the nozzle exit.

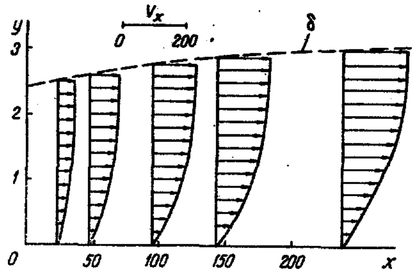

Fig. 2

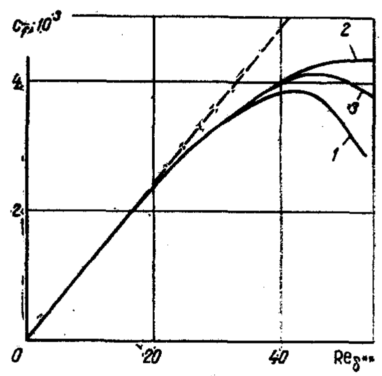

Fig. 3

Fig. 2. Distribution of longitudinal velocity and component over the obstacle.

Fig. 3. Distribution of friction coefficient over the obstacle: 1, 2, 3) calculation of c_f using 2, 3, and 4 terms of the series, respectively (the broken line is the calculation for uniform flow).

The calculation was carried out for the following specific data: $V_a = 8$ m/sec, $B_a = 0.1$ m, distance from the nozzle exit to the obstacle $Y_0 = 8B_a$, $\nu = 1.5 \cdot 10^{-5}$ m^2/sec. The velocity gradient at the stagnation point $\beta = V_m/V_\infty$ can be determined by solving the problem of inviscid interaction of the jet and the obstacle or can be evaluated from experimental data. In this analysis we used the approximation [4]:

$$\beta = \frac{9V_a}{\left(\frac{Y_0}{B_a}\right)^{1.2} B_a}.$$

For the given value of β, using the known dependence for the velocity distribution along the axis of a free jet V_m [5], one can easily determine Y_∞ and, therefore, the value of $X_{0,5}$ appearing in the series coefficients. However, calculations show that for the free section of the jet one can assume that $Y \simeq Y_0$ in determining $X_{0.5}$, with a sufficient accuracy.

The results of the solution of the system (10) are shown in Fig. 1a, b and in Table 1. From the results velocity profiles along the obstacle have been constructed for a series of values of x (Fig. 2), and the distribution of the friction coefficient at the wall $c_f = 2\tau_w(X)/\rho V_a^2$ has been given as a function of $Re_{\delta^{**}} = V_{x\delta}\delta^{**}/\nu$, calculated in terms of the momentum loss thickness δ^{**} (Fig. 3). Figure 3 also shows for comparison calculated friction values for flow of a uniform stream over an obstacle; c_f was calculated for a jet with successively 2, 3, and 4 terms of the series in Eq. (12).

It follows from the data presented that the effect of a nonuniformity of the flow on the flow field near the obstacle begins to be seen at very small distances from the stagnation point.

NOTATION

X	is the coordinate along the obstacle;
Y	is the direction normal to the obstacle;
B_a	is the width of the nozzle exit section;
Y_0	is the distance from the nozzle exit to the obstacle;
V_X and V_Y	are the velocity components along the X and Y axes, respectively;
V_a	is the velocity at the nozzle exit section;
Ω	is the degree of vorticity;
τ_w	is the friction stress;
c_f	is the coefficient of friction;
β	is the velocity gradient in the vicinity of the stagnation points;
ν	is the kinematic viscosity;
a	is the experimental constant.

Subscripts

w	at the obstacle;
m	along the jet axis;

a at the nozzle exit section;

δ at the outer edge of the wall layer.

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